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03 Hours / 70 Marks Seat No. [redacted]

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Use of Non-programmable Electronic Pocket Calculator is permissible.

(4) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. **Solve any FIVE of the following:** **10**

a) Evaluate : $\int\left(\frac{1}{1+x^2}+\cos x\right)dx$ $\Rightarrow$ $I = \tan^{-1}x + \sin x + C$

b) Evaluate : $\int\sqrt{1+\cos 2x}\ dx$ $\cos^2 x = \frac{1+\cos 2x}{2}$ $\therefore 2\cos^2 x = 1+\cos 2x$

$I = \int\sqrt{2\cos^2 x}\,dx \Rightarrow \sqrt{2}\cdot\sin x + C$

c) Evaluate : $\int_0^4 (4x-x^2)\ dx$ $\int_0^4 4x - x^2 = 32 - 64/3 = \frac{32}{3}$

d) Find the order and degree of the following differential equation $\frac{d^2y}{dx^2} = \sqrt{y-\frac{dy}{dx}}$ order = 2 degree = 2

e) Show that the root of the equation $x^3 - 2x - 5 = 0$ lies between 2 and 3. $f(2) = -1$ $f(3) = 16$ $x=2,\ x=3$. (IVT)

f) Find the approximate square root of a number 10 using Bakhshali Iterative method. $3^2 = 9$ $d = N - a^2$ $m = d/2a$ $x = a + m - \frac{m^2}{2a}$ $x \simeq 3.16$

g) A fair coin is tossed 8 times. Find the probability of getting exactly 2 heads.

Binomial Distribution –

$P(X=k) = \binom{n}{k}p^k(1-p)^{n-k}$

$\therefore n=8, k=2, p=0.5$

$P(X=2) = 28 \cdot 0.25 \cdot 0.0156$

$P(X=2) = 0.1093$

$P(X=2) = 10.94\%$

P.T.O.

a) $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \Rightarrow y = \sin^{-1}x \Rightarrow x = \sin y$ & $dx = \cos y\, dy$

$\sqrt{1-x^2} = \cos y$ & $\sin^{-1}x = y \Rightarrow \int \frac{1}{y^2}dy = -\frac{1}{y} + c \Rightarrow -\frac{1}{\sin^{-1}x} + c$

Marks

2. Solve any **THREE** of the following: 12

a) Evaluate : $\int \frac{1}{\sqrt{1-x^2}\,(\sin^{-1}x)^2}\,dx$

b) Evaluate : $\int \frac{\cos x}{(\sin x+1)(\sin x+2)}\,dx$

c) Evaluate : $\int e^x.\sin x\,dx$

d) Evaluate : $\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$

b) $u = \sin x \Rightarrow du = \cos x\, dx$

$\frac{1}{(u+1)(u+2)} = \frac{A}{u+1} + \frac{B}{u+2} \Rightarrow 1 = A(u+2) + B(u+1)$

$1 = Au + 2A + Bu + B \Rightarrow 1 = (A+B)u + (2A+B)$

coeff. $u = A+B = 0$, cost. $2A+B = 1$

$B = -A$, $A = 1$. $I = \int \left(\frac{1}{u+1} - \frac{1}{u+2}\right) du$

$\Rightarrow I = \int \left|\frac{u+1}{u+2}\right| + c \quad I = \int \left|\frac{\sin x+1}{\sin x+2}\right| + c$

3)

$u = \sin x$

$du = \cos x\, dx$

$du = e^x dx$

$v = e^x$

$\Rightarrow e^x \sin x - \int e^x \cos x\, dx$

$I_1 = \int e^x \cos x\, dx$

$u = \cos x$

$du = -\sin x\, dx$

$v = e^x$

$dv = e^x dx$

$\Rightarrow I_1 = e^x \cos x + \int e^x \sin x\, dx$

$\Rightarrow I = e^x \sin x - I_1$

$I = e^x \sin x - e^x \cos x - I$

$2I = e^x(\sin x - \cos x)$

$I = \frac{e^x(\sin x - \cos x)}{2} + c$

4) Quadratic part $-x^2 - 6x$

$16 - 6x - x^2 = 16 - (x^2 + 6x)$

$\Rightarrow x^2 + 6x = (x+3)^2 - 9$

$\Rightarrow 16 - 6x - x^2 = 16 - (x+3)^2 - 9 \Rightarrow 16 + 9 - (x+3)^2$

$\Rightarrow 25 - (x+3)^2$

$I = \int \frac{1}{\sqrt{25-(x+3)^2}}\,dx$

$u = x+3 \Rightarrow du = dx \Rightarrow I = \int \frac{1}{\sqrt{25-u^2}}\,du$

$I = \arcsin \frac{u}{5} + c$

3. Solve any **THREE** of the following: 12

a) Evaluate : $\int_0^{\pi/2} \frac{dx}{5+4\cos x}$

b) Evaluate : $\int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$

c) Solve the differential equation $(2xy + y^2)\,dx + (x^2 + 2xy + \sin y)\,dy = 0$

d) Using Bisection method find the root of the equation $x^3 - x - 1 = 0$ (Three iterations only)

a) $t = \tan\left(\frac{x}{2}\right)$

$\cos x = \frac{1-t^2}{1+t^2} \quad dx = \frac{2dt}{1+t^2}$

$I = \int_0^\infty \frac{2dt/1+t^2}{5+4\left(\frac{1-t^2}{1+t^2}\right)}$

$= \int_0^\infty \frac{2dt}{(1+t^2)\,5+4\left(\frac{1-t^2}{1+t^2}\right)}$

$I = \int_0^\infty \frac{2dt}{9+t^2}$

$\int \frac{dt}{a^2+t^2} = \frac{1}{a}\arctan\left(\frac{t}{a}\right)$

$a = 3$

$I = \frac{2}{3}\left[\arctan\left(\frac{t}{3}\right)\right]_0^\infty$

$I = 2/3\left(\frac{\pi}{2} - 0\right) = \pi/3$

c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$2x + 2y = 2x + 2y$

Potential fu^n.

$\frac{\partial \phi}{\partial x} = M(x,y) = 2xy + y^2$

$\frac{\partial \phi}{\partial y} = N(x,y) = x^2 + 2xy + \sin y$

$\phi(x,y) = \int (2xy + y^2)\,dx$

$= x^2 y + y^2 x + g(y)$

$\frac{\partial \phi}{\partial y} = x^2 + 2xy + g'(y)$

$\therefore x^2 + 2xy + \sin y$

$g'(y) = \sin y$

$g(y) = -\cos y + c$

$\phi(x,y) = x^2 y + y^2 x - \cos y + c$

$\phi(x,y) = c$

d) $f(x) = 0$

$f(1) = -1$, $f(2) = 5$

midpoint $= c = \frac{a+b}{2}$

Interation I

$a = 1$, $b = 1$

$c = 1.5$

$f(c) = f(1.5) = 0.875$ $[1, 1.5]$

Interation II

$a = 1$, $b = 1.5$ $c = 1.25$

$f(1.25) = -0.296$ $[1.25, 1.5]$

Interation III

$a = 1$, $b = 1.25$ $c = 1.375$

$f(1.375) = 0.224$

$[1.25, 1.375]$

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4.

4. **Solve any <u>THREE</u> of the following:** 12

~~a)~~ Find the root of the equation $x^3 + 2x^2 - 8 = 0$ using Regula Falsi method. (Perform three iterations)

~~b)~~ Using Newton Raphson method, find a root of the equation $x^4 - x - 9 = 0$, perform upto three iterations.

c) Solve the following equations by Gauss - Seidal method

$$5x - 2y + 3z = 18$$
$$x + 7y - 3z = 22$$
$$2x - y + 6z = 22$$

$x = \frac{18+2y-3z}{5}$ $y = \frac{22-x-3z}{7}$ $z = \frac{22-2x+y}{6}$

$x^{(0)}=0,\ y^{(0)}=0,\ z^{(0)}=0$

$x^{(1)} = 3.6$ $y^{(1)} = 2.62$ $z^{(1)} = 2.90$

~~d)~~ If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts drawn

i) One is defective

ii) at the most two are defective.

$x \approx 2.4261$
$y \approx 1.5346$
$z \approx 3.11$

~~e)~~ If probability that an electric motor is defective is 0.01, what is the probability that sample of 300 electric motors will contain exactly 5 defective motors?

5. **Solve any <u>TWO</u> of the following:** 12

a) ~~i)~~ Evaluate : $\int \frac{dx}{3-2\sin^2 x}$

~~ii)~~ Evaluate : $\int \frac{1-\tan x}{1+\tan x}\, dx$

b) ~~i)~~ Evaluate : $\int_0^1 \frac{dx}{x^2-x+1}$

~~ii)~~ Evaluate : $\int_0^{\pi/2} \sin^3 x \cos x\, dx$

c) ~~i)~~ Evaluate : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$

~~ii)~~ Evaluate : $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\, dx$

$I = \int \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\, dx \quad \text{---}(1)$

b)

$a+b = 0+4 = 4$

x by $4-x$

$I = \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{9-4+x}} \quad \text{---}(2)$

(1) + (2).

$2I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}} + \frac{\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\, dx$

$= \int_0^4 \frac{\sqrt[3]{x+5}+\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\, dx$

$2I = \int_0^4 1\, dx$

$= [x]_0^4$

$= [4-0]$

$2I = 4$

$\underline{I = 2}$

a) i) $y = ax^2 + b$
Diff. w.r.to. x
$dy/dx = a2x + 0$
$dy/dx = 2ax$

again diff. w.r.to x
$\frac{d^2y}{dx^2} = 2a$
$\frac{d^2y}{dx^2} - 2a = 0$

6. **Solve any TWO of the following:** **12**

ii) $\sec^2(\tan y)\, dx = -\sec^2(\tan x)\, dy$

a) i) Form the D.E. if $y = ax^2 + b$

ii) Solve : $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\frac{dx}{\sec^2(\tan x)} = -\frac{dy}{\sec^2(\tan y)}$

b) i) Solve the differential equation

$$x \frac{dy}{dx} + y = x^3$$

$\frac{dx}{\sec^2(\tan x)} = -\frac{dy}{\sec^2(\tan y)}$

$\because \frac{1}{\sec^2(\theta)} = \cos^2\theta$

ii) Show that the equation

$(3x^2 + 6xy^2)\, dx + (6x^2y + 4y^2)\, dy = 0$ is an exact D.E.

$\tan x + \frac{\sin(2\tan x)}{2} = -\left(\tan y + \frac{\sin(2\tan y)}{2}\right) + 2c$

$\frac{dy}{dx} = x^2 - \frac{y}{x}$

$x\frac{dy}{dx} + y = x^3$

$\frac{d}{dx}(xy) = x^3$

$\int \frac{d}{dx}(xy)\, dx = \int x^3 \, dx$

$\int x^3 dx = \frac{x^4}{4} + c$

$xy = \frac{x^4}{4} + c$

$y = \frac{x^3}{4} + \frac{c}{x}$

c) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

i) How many students score between 12 and 15?

ii) How many students score above 18?

Given:

A(0.8) = 0.2881

A(0.4) = 0.1554

A(1.6) = 0.4452

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c) $n = 1000$
$\mu = 14, \sigma = 2.5$
standard Normal curve

$z = \frac{x - \mu}{\sigma} = \frac{x - 14}{2.5}$ —(1)

a) for $x = 12$

$z = \frac{12 - 14}{2.5} = \frac{-2}{2.5} = -0.8$

$x = 15$, $z = 0.4$

$P(12 \le x \le 15) = P(-0.8 \le z \le 0.4)$

$= P(-0.8 \le z \le 0) + P(0 \le z \le 0.4)$

$= 0.2881 + 0.1554$

$= 0.443$

≈ 444

b) $x = 18$, $z = 1.6$

$P(x \ge 18) = P(z \ge 1.6)$

$= 0.5 - P(0 \le z \le 1.6)$

$= 0.0548$